

### 3. TESZT

#### I. Feladatsor

- 1/  $15 - 15 : 5 = 15 - 3 = \boxed{12}$
- 2/  $1000 \cdot 50\% = 1000 \cdot \frac{50}{100} = \boxed{500}$
- 3/  $-3 \cdot (-2) \cdot (-1) \cdot 0 \cdot 1 \cdot 2 = \boxed{0}$
- 4/  $K\ddot{n}egyzet = 8\text{cm} \Rightarrow a = 8 : 4 \Rightarrow \boxed{a = 2}$
- 5/  $m(\widehat{A'B'C}) = m(\widehat{ABC}) = \boxed{60^\circ}$
- 6/  $8 - 3 = \boxed{5}$  -tel többet adott.

#### II Feladatsor:

$$\begin{aligned} 2/ \quad a &= \sqrt{2} \cdot \left( \frac{\sqrt{2}}{2} + \sqrt{2} \right) + 2 = \sqrt{2} \cdot \left( \frac{\sqrt{2}}{2} + \sqrt{2} \right) + 2 = \\ &= \sqrt{2} \cdot \left( \frac{\sqrt{2}}{2} + \frac{2\sqrt{2}}{2} \right) + 2 = \sqrt{2} \cdot \frac{3\sqrt{2}}{2} + 2 = \frac{3 \cdot \sqrt{2} \cdot \sqrt{2}}{2} + 2 = \\ &= \frac{3 \cdot 2}{2} + 2 = 3 + 2 = 5 \Rightarrow \underline{a = 5} \\ b &= \sqrt{3} \cdot \left( \frac{\sqrt{3}}{3} - \sqrt{3} \right) + 4 = \sqrt{3} \cdot \left( \frac{\sqrt{3}}{3} - \frac{3\sqrt{3}}{3} \right) + 4 = \\ &= \sqrt{3} \cdot \left( \frac{\sqrt{3}}{3} - \frac{3\sqrt{3}}{3} \right) + 4 = \sqrt{3} \cdot \frac{-2\sqrt{3}}{3} + 4 = \\ &= \frac{-2 \cdot 3}{3} + 4 = -2 + 4 = 2 \Rightarrow \underline{b = 2} \end{aligned}$$

tehát  $a \neq b$ .

3/ Legyen a keresett számok  $a, b$ , és  $c$

$\{a, b, c\} \in \{3, 5, 7\}$ ;  $a$  legkisebb szám az  $\underline{a}$   
 $a$  legnagyobb szám az  $\underline{c}$

tehát  $a + c = 320$

$$\frac{a}{3} = \frac{b}{5} = \frac{c}{7} = k$$

$$\frac{a}{3} = k \Rightarrow a = 3k$$

$$\frac{c}{7} = k \Rightarrow c = 7k$$

$$\text{és } \frac{b}{5} = k \Rightarrow b = 5k$$

3. Teszt  
4/5

$$a+c=320 \text{ vagyis}$$

$$3k+7k=320 \Rightarrow 10k=320 \Rightarrow k=320:10$$

$$k=32$$

$$a=3k \Rightarrow a=3 \cdot 32 \Rightarrow a=96$$

$$b=5k \Rightarrow b=5 \cdot 32 \Rightarrow b=160$$

$$c=7k \Rightarrow c=7 \cdot 32 \Rightarrow c=224$$

A keresett számok 96, 160, 224.

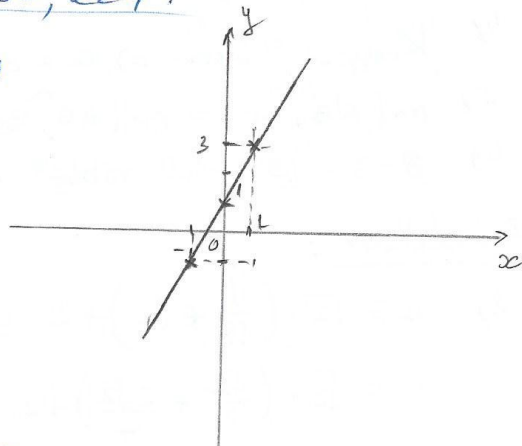
$$4) f: \mathbb{R} \rightarrow \mathbb{R}; f(x) = 2x+1$$

$$a) \begin{array}{c|ccc} x & -1 & 0 & 1 \\ \hline f(x) & -1 & 1 & 3 \end{array}$$

$$f(-1) = 2 \cdot (-1) + 1 = -2 + 1 = -1$$

$$f(0) = 1$$

$$f(1) = 2 \cdot 1 + 1 = 3$$



$$b) N = f(0) + f(1) + \dots + f(10) = 1 + 3 + 5 + 7 + \dots + 21$$

$$f(2) = 2 \cdot 2 + 1 = 5$$

$$f(3) = 2 \cdot 3 + 1 = 7$$

$$\vdots$$
$$f(10) = 2 \cdot 10 + 1 = 21$$

vagyis kihasználjuk az  $1+3+5+7+\dots+21$  összeget Gauss-módszerrel

$$\text{tehát } \underbrace{1+3+5+7+\dots+21}_{22} = \frac{22 \cdot 11}{2} = 11 \cdot 11 = 121$$

vagyis  $121 = 11^2$ , fellirható egy teljes négyzetként

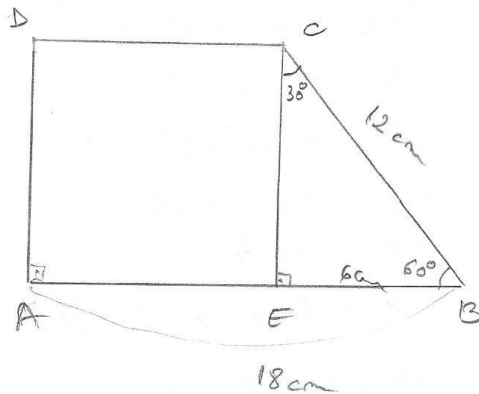
3. Teszt

2/5

$$\begin{aligned}
 5.) \quad E(x) &= \frac{x}{x^2+3x} - \left( \frac{x+3}{x-3} - \frac{x-3}{x+3} \right) : \frac{6}{x-3} = \\
 &= \frac{x}{x \cdot (x+3)} - \frac{x+3 - (x-3)}{(x-3)(x+3)} : \frac{6}{x-3} = \\
 &= \frac{x}{x \cdot (x+3)} - \frac{\cancel{x}+3 - \cancel{x}+3}{(x-3)(x+3)} : \frac{6}{x-3} = \\
 &= \frac{x}{x \cdot (x+3)} - \frac{6}{(x-3)(x+3)} \cdot \frac{x-3}{6} = \\
 &= \frac{x}{x \cdot (x+3)} - \frac{1}{x+3} = \\
 &= \frac{1}{x+3} - \frac{1}{x+3} = 0 \quad \Rightarrow \underline{E(x) = 0}
 \end{aligned}$$

III. Feladat sor:

- 1) F: ABCD trapéz  
 $m(\hat{A}) = 90^\circ$   
 $AB \parallel CD$   
 $AB = 18 \text{ cm}$   
 $CD = 12 \text{ cm}$   
 $m(\hat{ABC}) = 60^\circ$   
 $CE \perp AB, E \in (AB)$



- K: a)  $BE = 6 \text{ cm}$   
 b)  $T_{ABCD} = ?$   
 c)  $H_C: AF = FE$   
 $CF \perp BD$

B: a)  $\Delta EBC$ -ben  $m(\hat{E}) = 90^\circ$   
 $m(\hat{B}) = 60^\circ$  }  $\Rightarrow m(\hat{C}) = 30^\circ$   
 $\Downarrow$   
 $EB = \frac{BC}{2}$

$\Rightarrow EB = \frac{12}{2} \Rightarrow \underline{EB = 6 \text{ cm}}$

3. rész  
 3/5

$$b) T_{ABCD} = \frac{(AB + DC) \cdot CE}{2}$$

$$DC = AE = AB - EB = 18 - 6 = 12 \Rightarrow DC = 12 \text{ cm}$$

$$T_{ABCD} = \frac{(18 + 12) \cdot CE}{2}$$

$$CEB_{\Delta} \text{-ben } \hat{m}(\hat{E}) = 90^{\circ}$$

$$CE^2 = BC^2 - EB^2 \Rightarrow CE^2 = 12^2 - 6^2 \Rightarrow CE^2 = 144 - 36 \Rightarrow CE^2 = 108$$

$$CE = \sqrt{108} \Rightarrow CE = 6\sqrt{3} \text{ cm}$$

$$T_{ABCD} = \frac{(18 + 12) \cdot 6\sqrt{3}}{2} = \frac{30 \cdot 6\sqrt{3}}{2} = 90\sqrt{3} \text{ cm}^2$$

$$c) DC = 12 \text{ cm}$$

$$FB = FE + EB = \frac{AE}{2} + EB = 6 + 6 = 12$$

$$FB = 12 \text{ cm}$$

$$ECB_{\Delta} \text{-ben } \hat{m}(\hat{E}) = 90^{\circ}$$

$$CB^2 = CE^2 + EB^2$$

$$CB^2 = (6\sqrt{3})^2 + 6^2$$

$$CB^2 = 36 \cdot 3 + 36$$

$$CB^2 = 36 \cdot 4$$

$$CB^2 = 144 \Rightarrow CB = \sqrt{144} \Rightarrow CB = 12 \text{ cm}$$

$$DAF_{\Delta} \cong CEB_{\Delta} \Rightarrow DF = BC = 12 \text{ cm}$$

$$\text{Tehát } DC = CB = FB = DF = 12 \text{ cm}$$

$$DC \parallel AB$$

}  $\Rightarrow$  DFBC négyoldal  
rombusz

$\Downarrow$

$$FC \perp BD$$

3. Tétel

4/5

2.)  $\frac{F}{ABCD A'B'C'D'}$  - kocka

$$AB = 10 \text{ cm}$$

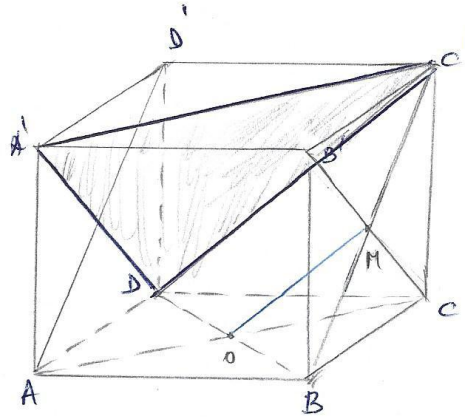
$$AC \cap BD = \{O\}$$

$$B'C \cap B'C' = \{M\}$$

k:  $\frac{1}{9} T_{ABCD} = 100 \text{ cm}^2$

b)  $d(D'; AB) = ?$

c)  $OM \parallel (C'DA')$



B: a)  $T_{ABCD} = AB^2 \Rightarrow T_{ABCD} = 10^2 \text{ cm}^2 \Rightarrow T_{ABCD} = 100 \text{ cm}^2$

b)  $d(D'; AB) = AD'$

$$\left. \begin{array}{l} D'D \perp (ABC) \\ AD \perp ABC \\ D'D \perp AD \\ AD \perp AB \end{array} \right\} \begin{array}{l} 3. \perp \\ \Rightarrow \\ \downarrow \end{array} \Rightarrow D'A \perp AB$$

$$\downarrow \\ d(D'; AB) = AD'$$

$D'AD_{\Delta}$  -ben  $m(\hat{D}) = 90^\circ$

$$D'A^2 = AD^2 + D'D^2$$

$$D'A^2 = 10^2 + 10^2$$

$$D'A^2 = 200 \Rightarrow \underline{D'A = 2\sqrt{10} \text{ cm}}$$

c)  $\star$   $DC'B_{\Delta}$  -ben

$$\left. \begin{array}{l} DO = OB; O \in (DB) \\ CH = MB; H \in (C'B) \end{array} \right\} \Rightarrow OM - \text{középvonal}$$

$$\downarrow \\ OM \parallel DC'$$

$$\left. \begin{array}{l} DC' \subset (C'DA') \\ DC' \parallel OM \end{array} \right\} \Rightarrow OM \parallel (C'DA')$$

3. írás  
5/5